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An advance on the theory of forced turbulent-flow film boiling heat transfer for subcooled liquid flowing along a horizontal flat plate

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Abstract—The semi-empirical theory for turbulent-flow film boiling of a subcooled liquid flowing with high velocity along a horizontal flat plate [*Int. J. Heat Mass Transfer* **28**, 1049–1055 (1985)] is further advanced in this paper. It is found that the empirical constant m can be predicted analytically as 2/3 for different fluids. The another empirical coefficient k is revealed to be a function of thermophysical properties of the liquid and its vapor, the actual degree of surface superheat and degree of liquid subcooling.

1. INTRODUCTION

A SEMI-empirical theory had been presented for the forced-flow turbulent film boiling of subcooled liquid along a horizontal plate [1], to suit the needs of developing efficient cooling-control technology of high-temperature quenching process, which is regarded as one of the more important technical duties in metallurgical industries. The physical model and analytical method proposed [1] have been extended successfully to cover the case of subcooled liquid flowing in a horizontal flat duct [2] and in a circular tube [3].

In the previous paper [1], the turbulent diffusivity of momentum exchange within the liquid flow region, $\varepsilon_{M,2}$, for the film boiling in violent turbulent flow of subcooled liquid, is expressed in a more general dimensionless form as:

$$\varepsilon_{M,2}/\nu_2 = k\,Re_x^m. \tag{1}$$

It has been pointed out that the dimensionless coefficient k may be a weak function of the thermophysical properties of liquid and can be taken as an empirical constant to be determined by experiments, while the exponent m depends on fluctuation of the liquid–vapor interface and could approach an empirical constant to be determined by experiments. The 'local revised Nusselt number', defined as

$$\widetilde{Nu}_x = \frac{q_{w,x}}{t_s - t_\infty}\frac{x}{\lambda_2} \tag{2}$$

can be predicted from the simplified analytical equation as

$$\widetilde{Nu}_x = k'\,Re_x^{(m+1)/2}\,Pr_2 \tag{3}$$

where

$$k' = \sqrt{\frac{k(m+1)}{\pi}}. \tag{4}$$

Equation (3) had been verified by experiments [1, 4] for deionized water flowing at atmospheric pressure, with velocity in the range 1–4.5 m s^{-1} and with water temperature in the subcooled range 22–72°C. The empirical values thus determined by the experimental data, taken from $x = 50$ and 150 mm, were $k = 0.0055$ and $m = 0.68$. Hence, equation (3) evolves to

$$\widetilde{Nu}_x = 0.054\,Re_x^{0.84}\,Pr_2. \tag{5}$$

Equation (3) was derived from the physical model suggested [1], the higher the flow velocity and the greater the degree of temperature subcooling for liquid deviated from its critical state, the more the basic assumptions tend towards reality. So, it is reasonable to expect that equation (3) will be suitable for the turbulent film boiling on a horizontal plate for the subcooled liquid flow, and equation (5) can be extended to predict the heat transfer rate in the high-temperature quenching process of rolled metal sheets, especially for water with a greater degree of temperature subcooling and flowing at a higher velocity [1]. It should be emphasized that $k = 0.0055$ and $m = 0.68$ were obtained from experiments with subcooled water only [2]. However, as will be shown, the value of m can be predicted analytically as a universal constant, 2/3, for different liquids.

2. FUNDAMENTAL CONSIDERATIONS

The physical model adapted is retained unchanged [1], as shown in Fig. 1.

The vapor–liquid mixing process in the intermediate region, composed of vapor bubble flow and fluctuating liquid–vapor interface, is expected to be so strong that not only the time-mean velocity distribution within the mixing region will be uniform, but also the time-mean temperature within the region can be considered as being constant and equal to the saturated temperature, t_s.

NOMENCLATURE

a	thermal diffusivity
c_p	specific heat at constant pressure
h_{fg}	latent heat of evaporation
Nu	Nusselt number
Pr	Prandtl number
q	local heat transfer rate per unit area
Re	Reynolds number
t	temperature
u, v	velocity component in x and y directions, respectively.

Greek symbols

α	local heat transfer coefficient
δ	thickness of vapor film
λ	thermal conductivity
ρ	density
μ	absolute viscosity
ν	kinematic viscosity.

Subscripts

w	at surface
∞	at infinite distance from plate surface
s	saturated condition
1	vapor
2	liquid.

The time-mean velocity distribution within the liquid flow region is uniform and kept constant as free-stream velocity, u_∞, and the turbulent boundary layer would be thus limited to within the vapor-film layer, because the time-mean velocity distribution in the vapor–liquid mixing region has already been assumed to be uniform and will be equal to u_∞ also. This confirms the result reported by Ito and Nishikawa [5] who solved numerically two-phase boundary equations. The flow characteristics in the vapor-film region will be the same as that of single-phase turbulent vapor flow along a horizontal plate surface. Therefore, as observed by Bradfield [6], the wall friction for turbulent-flow film boiling is obviously lower than for a single-phase turbulent liquid flow, since viscosity of vapor is much smaller than that of liquid.

It is found from numerical analysis on the basic mathematical formulation presented in ref. [1], that the rate of increase in δ_x will diminish quickly along x. So, we assume

$$\delta_x = b x^n \tag{6}$$

b is a dimensional coefficient.

For the forced-turbulent film boiling, the increase in vapor-film thickness along the flow direction will intensify the phase-transition process within the liquid–vapor mixing region, meanwhile, fluctuations at the vapor–liquid interface as well as vortex movement of liquid will spread rapidly into the liquid region, so that the turbulent viscosity, $\varepsilon_{M,2}$, becomes very large and approximately uniform at any given x-section, i.e. $\varepsilon_{M,2}$ may be independent of y at $y \geqslant \delta'_x$. To this end, instead of equation (1), we take for the liquid flow region

$$\frac{u_\infty \delta_x}{\varepsilon_{M,2}} = \text{constant}, \quad \text{or } \varepsilon_{M,2} = c u_\infty \delta_x. \tag{7}$$

In addition, the turbulent film boiling for liquid subcooled greatly, the increase of vapor-film thickness is strongly suppressed [8]. Especially, for the case of subcooled liquid flowing with high velocity along horizontal plate surface, the vapor-film thickness δ_x itself would be small. Hence, referring to Fig. 1, the heat flux transferred to the vapor film from the plate surface can be assumed to be almost completely transferred to the liquid flow, i.e.

$$q_{w,x} \approx q_{1,x}. \tag{8}$$

This is equivalent with the assumed temperature of vapor dropping sharply from $t_{w,x}$ to t_s, which thus results in a very high heat transfer rate.

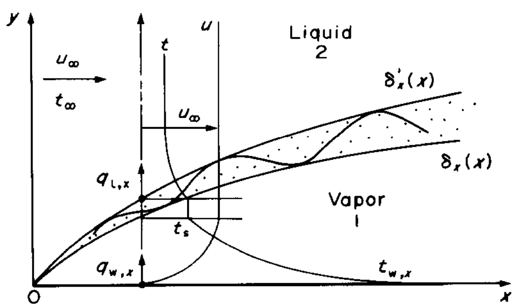

FIG. 1. Analytical model.

3. THEORETICAL ANALYSIS

The energy equation in the liquid flow region should be:

$$u_\infty \frac{\partial t_2}{\partial x} = \frac{\partial}{\partial y}\left[(a_2 + \varepsilon_{t,2})\frac{\partial t_2}{\partial y}\right] \tag{9}$$

where t_2 is the time-mean temperature of liquid, $\varepsilon_{t,2} = \lambda_{t,2}/(\rho_2 c_{p2})$ is the turbulent thermal diffusivity and $\lambda_{t,2}$ is the turbulent thermal conductivity of liquid.

Noting, $\varepsilon_{t,2} \gg a_2$, and $\lambda_{t,2} \gg \lambda_2$ for the violent turbulent liquid flow. Taking conventionally

$Pr_t = \varepsilon_M/\varepsilon_t \approx 1$ [7], where ε_M is the turbulent viscosity for momentum exchange, equation (9) can be simplified to

$$u_\infty \frac{\partial t_2}{\partial x} = \frac{\partial}{\partial y}\left(\varepsilon_{M,2}\frac{\partial t_2}{\partial y}\right). \tag{10}$$

For the case $\varepsilon_{M,2}$ assumed independent of y, equation (10) evolves to

$$u_\infty \frac{\partial t_2}{\partial x} = \varepsilon_{M,2}\frac{\partial^2 t_2}{\partial y^2}. \tag{11}$$

Introducing a dimensionless subcooled temperature, $\vartheta = (t_s - t_2)/(t_s - t_\infty)$, and substituting equation (7) into equation (11), changing the co-ordinate referring to the vapor–liquid interface, i.e. taking $y' = 0$ at $y = \delta'_x$, the following equation is obtained:

$$\frac{\partial \vartheta}{\partial x} = bcx^n \frac{\partial^2 \vartheta}{y'^2}. \tag{12}$$

Let

$$\eta = x^{n+1} \tag{13}$$

then

$$\frac{\partial \vartheta}{\partial x} = (n+1)x^n \frac{\partial \vartheta}{\partial \eta}$$

and equation (12) transforms to

$$\frac{\partial \vartheta}{\partial \eta} = \left(\frac{bc}{n+1}\right)\frac{\partial^2 \vartheta}{\partial y'^2} \tag{14}$$

with boundary conditions:

$$\left.\begin{array}{ll} x = 0, & \vartheta = 1 \\ y' = 0, & \vartheta = 0 \\ y' = \infty, & \vartheta = 1. \end{array}\right\} \tag{15}$$

The solution of equations (14) and (15) is given as:

$$\vartheta = \operatorname{erf}\left[\frac{y'}{2x^{(n+1)/2}\sqrt{bc/(n+1)}}\right]. \tag{16}$$

Hence, from Fourier's law,

$$q_{1,x} = \rho_2 c_{p2}\varepsilon_{M,2}(t_s - t_\infty)\left.\frac{\partial \vartheta}{\partial y'}\right|_{y'=0}$$

or (17)

$$q_{1,x} = \rho_2 c_{p2}(t_s - t_\infty)\sqrt{\frac{bc(n+1)}{\pi}}\,x^{(n-1)/2}.$$

For the vapor-film region, both Karman's universal velocity distribution and the analogy between momentum and heat transfer would hold true, together with the boundary conditions

$$\left.\begin{array}{ll} y = 0, & t_1 = t_{w,x} \\ y = \delta_x, & t_1 = t_s \end{array}\right\} \tag{18}$$

the following familiar correlation can be obtained as [7]:

$$\frac{t_{w,x} - t_s}{q_{w,x}} = \frac{u_{\delta_x}\Big/\sqrt{s_{w,x}/\rho_1} + 5(Pr_1 - 1)\ln\left(\dfrac{5Pr_1 + 1}{6}\right)}{\rho_1 c_{p1}\sqrt{s_{w,x}/\rho_1}} \tag{19}$$

where $s_{w,x}$ is the shear stress on wall surface. Using Blasius' formula for the friction coefficient $c_{f,x}$ of single-phase flow, we get

$$s_{w,x} = \frac{1}{2}\rho_1 u_\infty^2 c_{f,x} = 0.023\left(\frac{u_\infty \delta_x}{\rho_1}\right)^{-1/4}\rho_1 u_\infty^2. \tag{20}$$

For most liquids, including water, deviating greatly from their critical states, $Pr_1 \approx 1$, equation (19) can be then reduced to

$$\frac{t_{w,x} - t_s}{q_{w,x}} = \frac{u_\infty}{c_{p1}s_{w,x}}. \tag{21}$$

Combine equations (20) and (21), then clearly

$$q_{w,x} = 0.023\rho_1 c_{p1} u_\infty (t_{w,x} - t_s)\left(\frac{u_\infty \delta_x}{\nu_1}\right)^{-1/4}. \tag{22}$$

According to equation (8), we obtain from equations (19) and (22),

$$\rho_2 c_{p2}(t_s - t_\infty)\sqrt{\frac{bc(n+1)}{\pi}}\,x^{(n-1)/2}$$
$$= 0.023\rho_1 c_{p1} u_\infty^{3/4}(t_{w,x} - t_s)\left(\frac{b}{\nu_1}\right)^{-1/4} x^{-n/4}. \tag{23}$$

The identity of equation (23) for any x requires the necessity of the same exponent of x in both sides, i.e.

$$\frac{n-1}{2} = -\frac{n}{4}, \quad \text{or } n = \frac{2}{3}. \tag{24}$$

Hence, from equation (23),

$$b = 0.01\left[\frac{\rho_1 c_{p1}(t_{w,x} - t_s)}{\rho_2 c_{p2}(t_s - t_\infty)}\right]^{4/3}\left(\frac{u_\infty}{\nu_1}\right)^{-1/3}\left(\frac{5c}{3\pi}\right)^{-2/3}. \tag{25}$$

Substituting equations (25) and (6) into equation (7), we have

$$\frac{\varepsilon_{M,2}}{\nu_2} = c'\left[\frac{\rho_1 c_{p1}(t_{w,x} - t_s)}{\rho_2 c_{p2}(t_s - t_\infty)}\right]^{4/3}\left(\frac{\nu_1}{\nu_2}\right)^{1/3}\left(\frac{u_\infty x}{\nu_2}\right)^{2/3} \tag{26}$$

where $c' = 0.01(5c/3\pi)^{-2/3}$ is a dimensionless empirical constant. It is clear that comparing equations (26) and (1)

$$m = 2/3, \tag{27}$$

and

$$k = c'\left[\frac{\rho_1 c_{p1}(t_{w,x} - t_s)}{\rho_2 c_{p2}(t_s - t_\infty)}\left(\frac{\nu_1}{\nu_2}\right)^{1/4}\right]^{4/3}. \tag{28}$$

From equation (4)

$$k' = c''\left(\frac{\mu_1}{\mu_2}\right)^{1/6}\left[\frac{\rho_1 c_{p1}(t_{w,x}-t_s)}{\rho_2 c_{p2}(t_s-t_\infty)}\right]^{2/3} \quad (29)$$

where $c'' = 0.1(5c/3\pi)^{1/3}$. Then, equation (3) evolves to

$$\widetilde{Nu}_x = k' Re_x^{5/6} Pr_2. \quad (30)$$

The exponent of Re_x here is $5/6 = 0.8\dot{3}$, which coincides very well with the empirical value 0.84 in equation (5).

4. CONCLUDING REMARKS

The semi-empirical theory [1] for the turbulent film boiling heat transfer of a subcooled liquid, greatly deviating from its critical state and flowing with higher velocity along a horizontal flat plate, is further advanced.

It is predicted analytically that the value of m is 2/3, i.e. a universal constant for different fluids, which coincides with the empirical value from the deionized water, reported as 0.68 [1, 4].

The heat transfer relation, equation (30) is very nearly the same as that reported in [1, equation (5)]. The analysis presented here, however, reveals that the value of k, and then the value of k', is not only a function of thermophysical properties of the liquid and its vapor, but is also affected by the actual degree of surface superheat, $(t_{w,x}-t_s)$, and degree of liquid subcooling, (t_s-t_∞). So k and k' will not really be constant even for the same liquid flowing along the flat plate. The empirical values $k = 0.0055$ and $k' = 0.054$ were obtained empirically from specified experiments with subcooled water [1, 4]. Further experiments will still be necessary to check the suitability of equation (29) or equation (28) as well as to determine the value of c'' or $c' = (c'')^2$ for different liquids.

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SUR LA THEORIE DU TRANSFERT THERMIQUE POUR UN FILM LIQUIDE SOUS-REFROIDI EN EBULLITION AVEC ECOULEMENT TURBULENT FORCE LE LONG D'UNE PLAQUE HORIZONTALE

Résumé—On améliore la théorie semi-empirique de l'ébullition en film turbulent d'un liquide sous-refroidi qui s'écoule à grande vitesse sur une plaque plane horizontale [*Int. J. Heat Mass Transfer* **28,** 1049–1055 (1985)]. On trouve que la constante empirique m peut être prédéterminée analytiquement comme égale à 2/3 pour différents fluides. L'autre coefficient empirique k se révèle être une fonction des propriétés thermophysiques du liquide et de sa vapeur, du degré de surchauffe de la surface et du degré de sous-refroidissement du liquide.

WÄRMEÜBERGANG BEIM FILMSIEDEN IN ERZWUNGENER, TURBULENTER UND UNTERKÜHLTER FLÜSSIGKEITSSTRÖMUNG

Zusammenfassung—Die halbempirische Theorie für das Filmsieden bei turbulenter Strömung einer unterkühlten Flüssigkeit bei großen Geschwindigkeiten entlang einer horizontalen Platte [*Int. J. Heat Mass Transfer* **28,** 1049–1055 (1985)] wurde weiterentwickelt. Man findet, daß die empirische Konstante *m* analytisch mit 2/3 für verschiedene Fluide angegeben werden kann. Der andere empirische Koeffizient *k* ist eine Funktion der thermophysikalischen Eigenschaften der Flüssigkeit und des Dampfes, sowie der Wandüberhitzung und der Flüssigkeitsunterkühlung.

РАЗВИТИЕ ТЕОРИИ ТЕПЛООБМЕНА ПРИ ПЛЕНОЧНОМ КИПЕНИИ В ВЫНУЖДЕННОМ ТУРБУЛЕНТНОМ ПОТОКЕ ДЛЯ НЕДОГРЕТЫХ ЖИДКОСТЕЙ, ДВИЖУЩИХСЯ ВДОЛЬ ПЛОСКОЙ ГОРИЗОНТАЛЬНОЙ ПЛАСТИНЫ

Аннотация—Развивается полуэмпирическая теория пленочного кипения в турбулентном потоке недогретой жидкости, движущейся с большой скоростью вдоль плоской горизонтальной пластины [*Int. J. Heat and Mass Transfer* **28**, 1049–1055 (1985)]. Аналитически рассчитано, что эмпирическая постоянная m равна 2/3 для различных жидкостей. Найдено, что другой эмпирический коэффициент k является функцией теплофизических свойств жидкости и ее пара, фактической степени перегрева поверхности и степени недогрева жидкости.